PROJECT MANAGEMENT

Topic 3

Probabilistic Activity Times

PERT

Contents

Probabilistic Activity Times

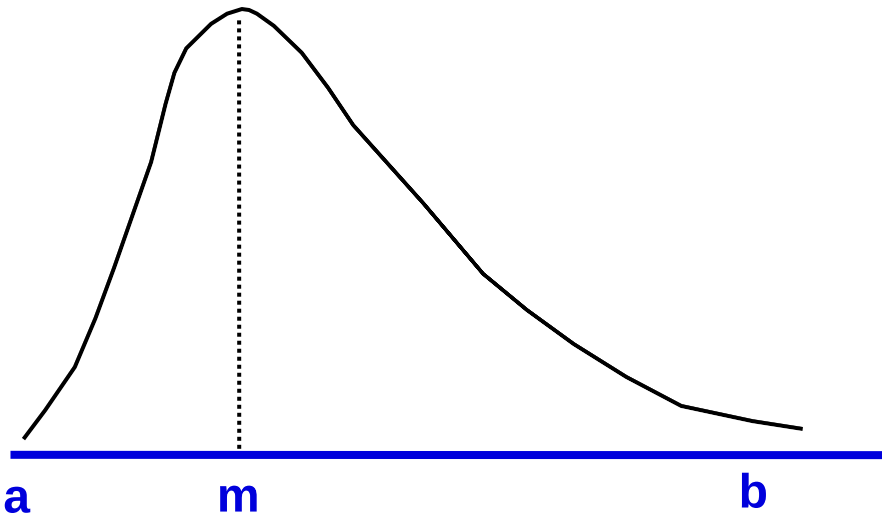
- Activity time estimates usually cannot be made with certainty.
- ***PERT used for probabilistic*** activity times.
- In PERT, three time estimates are used: ***most likely*** time (m), the ***optimistic*** time (a), and the ***pessimistic*** time (b).
- These provide an estimate of the ***mean and variance*** of a beta distribution:

Variance: $v = \left(\frac{b - a}{6}\right)^2$

Mean (expected time)

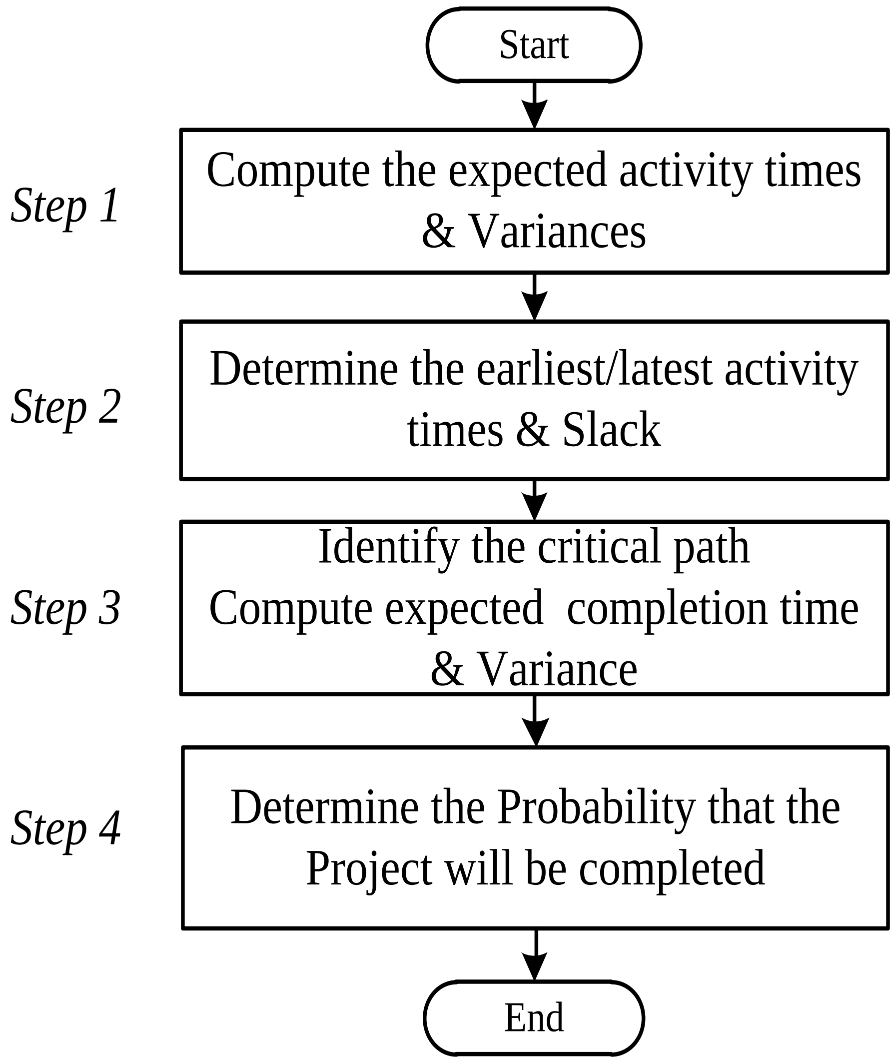
$$t = \frac{a + 4m + b}{6}$$

Three-Point Method



Probabilistic Activity Times

Probability Analysis Process



Probabilistic Activity Times

House Building project data

No	Time estimation (mo)			Time t	Variant V
	a	m	b		
1. Design house & Obtain financing	2	3	4	3	$1/9^*$
2. Lay foundation	1	2	3	2	$1/9^*$
3. Order Materials	0.5	1	1.5	1	1/36
4. Build house	2	3	4	3	$1/9^*$
5. Select paint	0.5	1	1.5	1	1/36
6. Select carpet	0.5	1	1.5	1	1/36
7. Finish work	0.5	1	1.5	1	$1/36^*$

Table 3.1. *Activity Time Estimates*

Probabilistic Activity Times

Critical path: 1- 2 – 4 -7

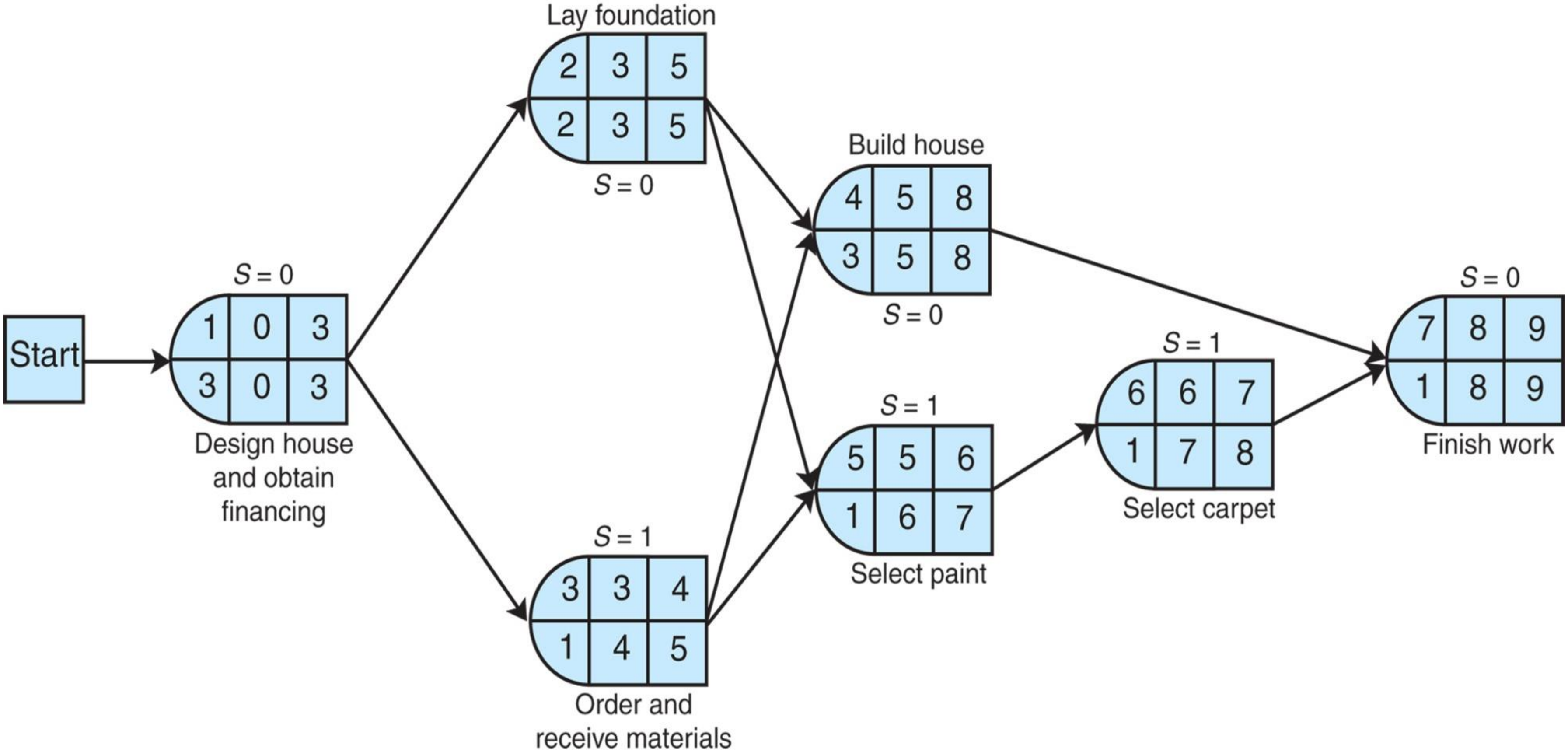


Figure 3.1. *Activity Slack*

Probabilistic Activity Times

Expected Project Time and Variance

- ***Expected project time*** is the sum of the expected times of the critical path activities.
- ***Project variance*** is the sum of the critical path activities' variances
- The expected project time is ***assumed to be normally distributed*** (based on central limit theorem).
- In example, expected project time (t_p) and variance (v_p) interpreted as the mean (μ) and variance (σ^2) of a normal distribution:

μ = 9 Month

$\sigma^2 = 13/36\ (\text{Month})^2$

Critical Path Activity	Variance
1	1/9
2	1/9
4	1/9
7	1/36

Probability Analysis of a Project Network

- The normal distribution, probabilities are determined by computing standard deviations value (Z).
- The Z value is used to find corresponding probability.

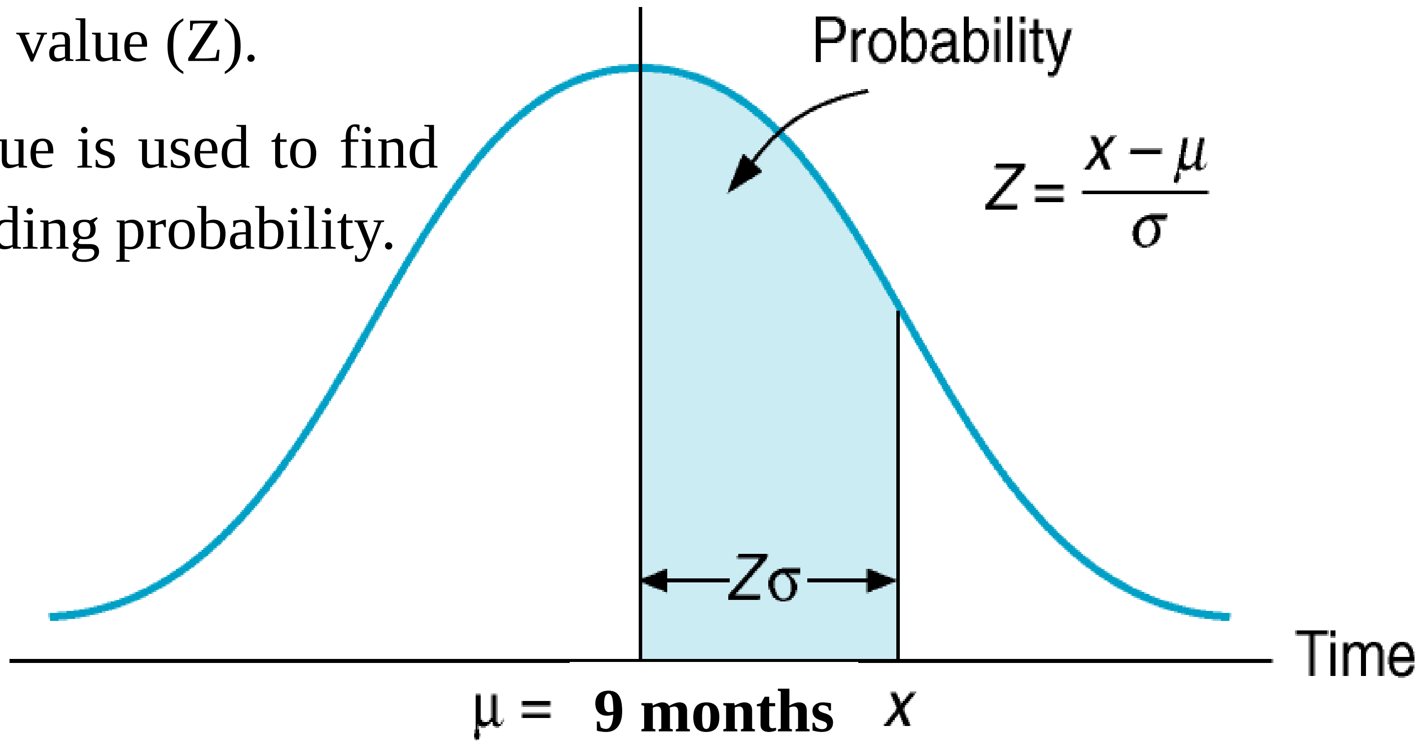


Figure 3.2. *Normal Distribution of Network Duration*

Probability Analysis of a Project Network

What is the probability that the new order processing system will be ready by 10 months?

$\mu = 9$ months

$\sigma^2 = 0.36 \quad \sigma = 0.6$ months

$Z = (x-\mu)/\sigma = (10 - 9)/0.6 = 1.67$

Z value of 1.67 corresponds to probability of 0.9525 in Table 3.2.
Probability of completing project in 10 months or less = 0.9525.

Probability Analysis of a Project Network

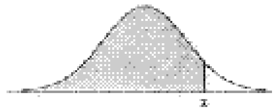


Table 3.2

Probability Content from -oo to Z

Z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990

Probability Analysis of a Project Network

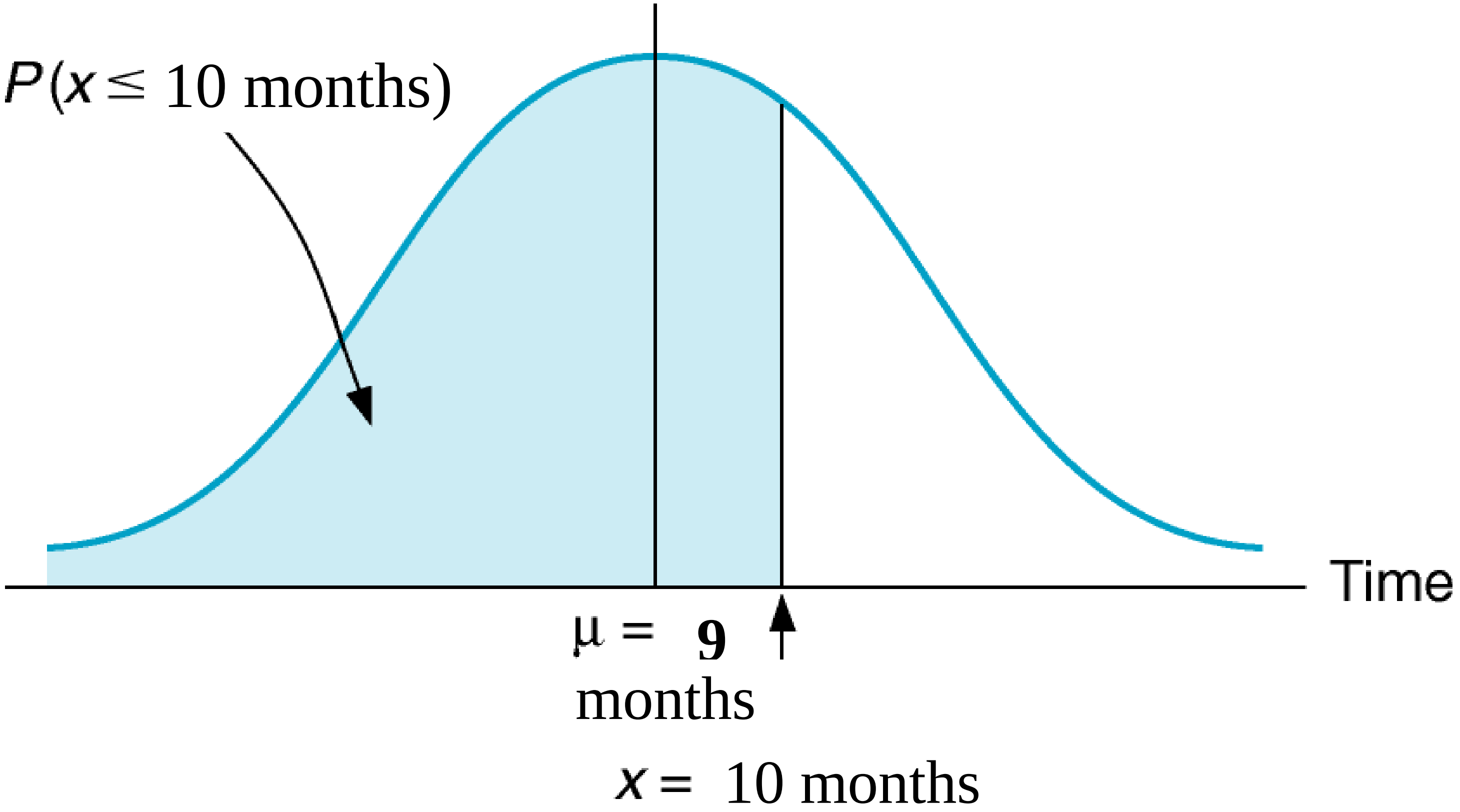


Figure 3.3. *Probability the Network will be completed in 10 months or less*

Probability Analysis of a Project Network

- A customer will trade elsewhere if the new ordering system is not working within 8 months. What is the probability that she will be retained?

$$Z = (8 - 9)/0.6 = -1.67$$

- Z value of 1.67 (ignore negative) corresponds to probability of .9525 in Table 3.2.
- Probability that customer will be retained is .0475

Probability Analysis of a Project Network

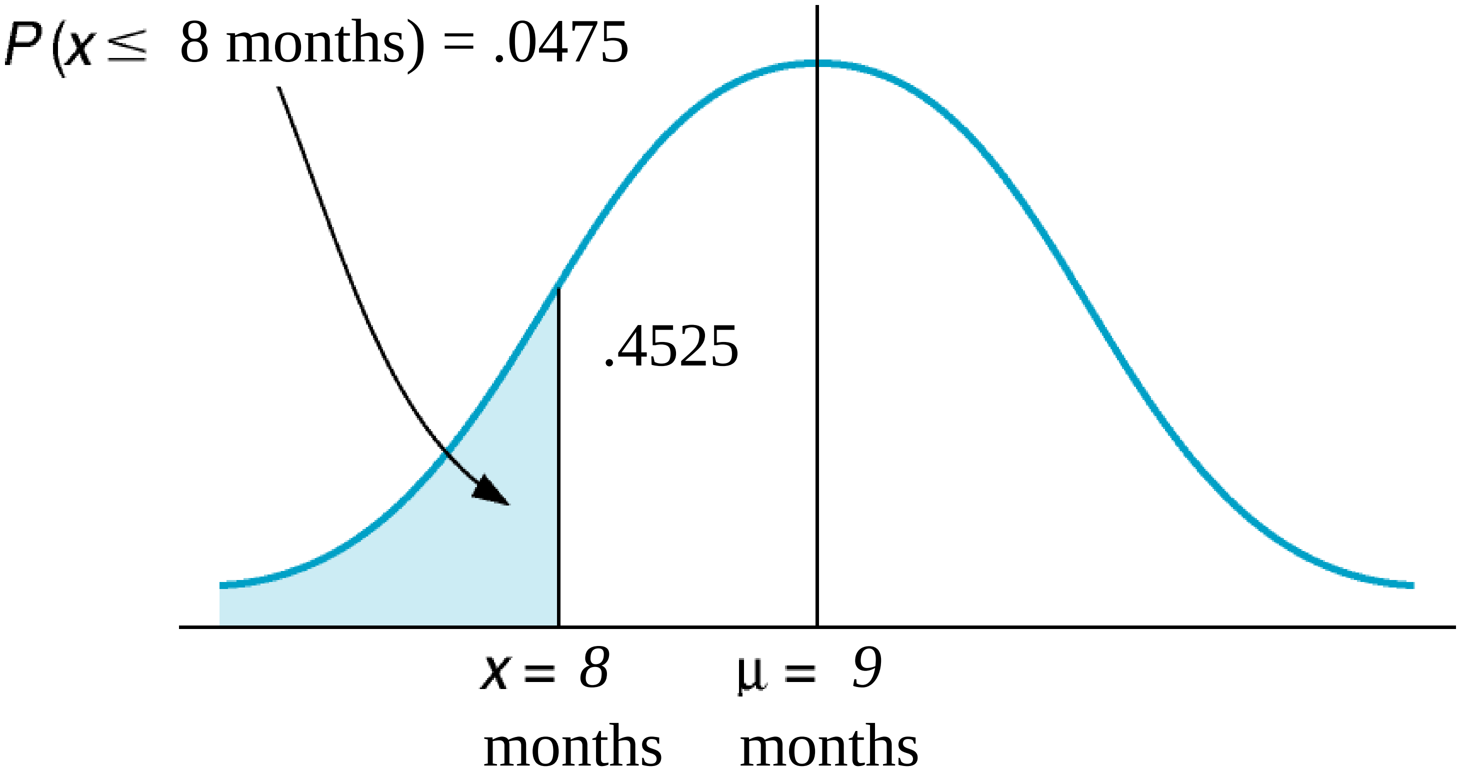


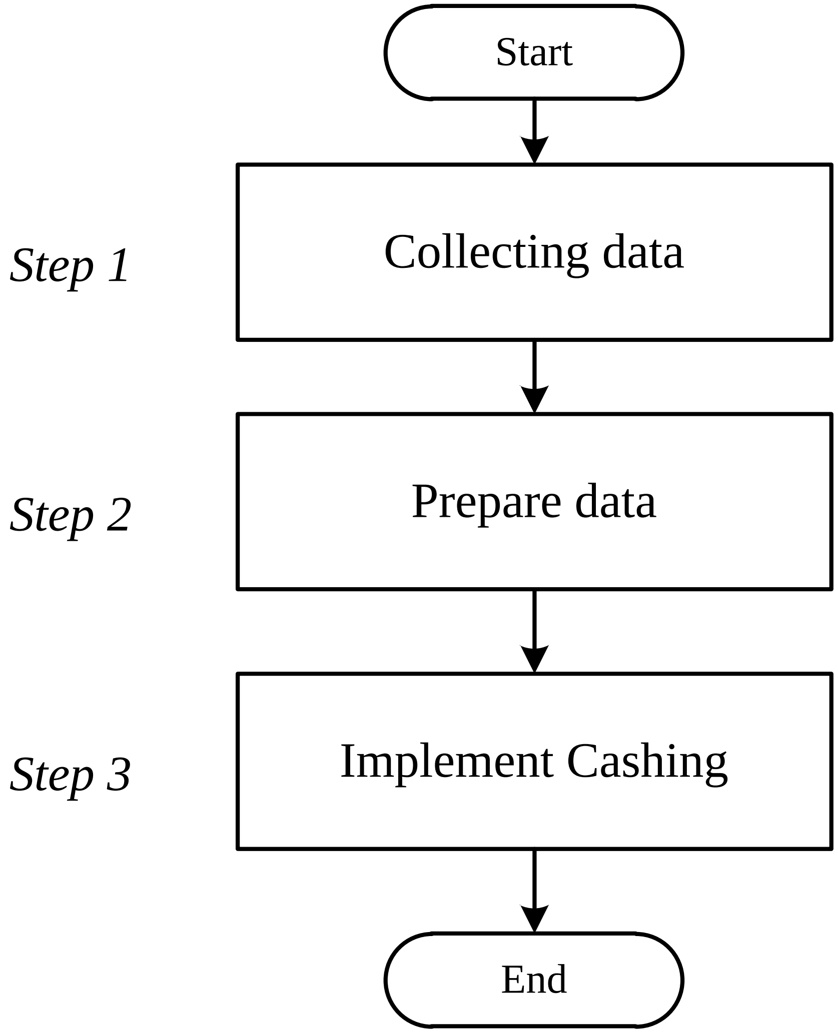
Figure 3.4. *Probability the Network will be completed in 8 months or less.*

Project Crashing

- Project ***duration can be reduced*** by assigning more resources to project activities.
- However, doing this ***increases project cost***.
- Decision is based on analysis of ***trade-off between time and cost***.
- ***Project crashing*** is a method for shortening project duration by reducing one or more ***critical activities*** to a time less than normal activity time.

Project Crashing

Cashing Process



1. Normal cost
2. Cash cost
3. Cash time

4. Determining Critical Path
5. Allowable Cash time
6. Cash cost per time

7. Cashing based on Required
8. Drawing Critical Path Subgraph

Project Crashing

Project network (week)

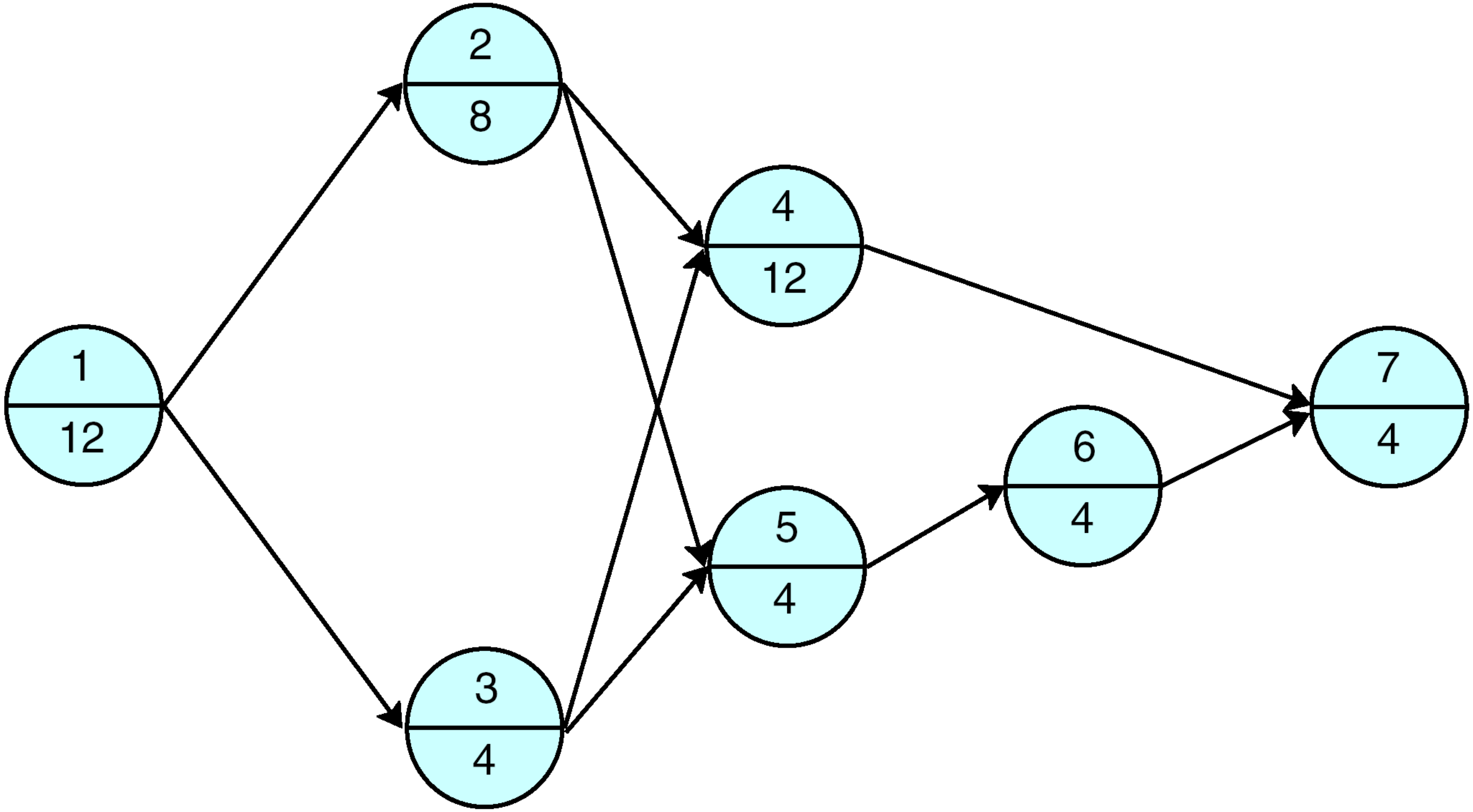


Figure 3.5. *The Project Network for Building a house*

Project Crashing

Prepare data

Activity	Normal Time (weeks)	Crash Time (weeks)	Normal Cost	Crash Cost	Total Allowable Crash Time (weeks)	Crash Cost per Week
1	12	7	$ 3,000	$ 5,000	5	$ 400
2	8	5	2,000	3,500	3	500
3	4	3	4,000	7,000	1	3,000
4	12	9	50,000	71,000	3	7,000
5	4	1	500	1,100	3	200
6	4	1	500	1,100	3	200
7	4	3	15,000	22,000	1	7,000
			$75,000	$110,700		

Project Crashing

Crash cost & crash time have a linear relationship:

$$\frac{Total\ Crash\ Cost}{Total\ Crash\ Time} = \frac{\$2000}{5\ weeks} = \$400 / wk$$

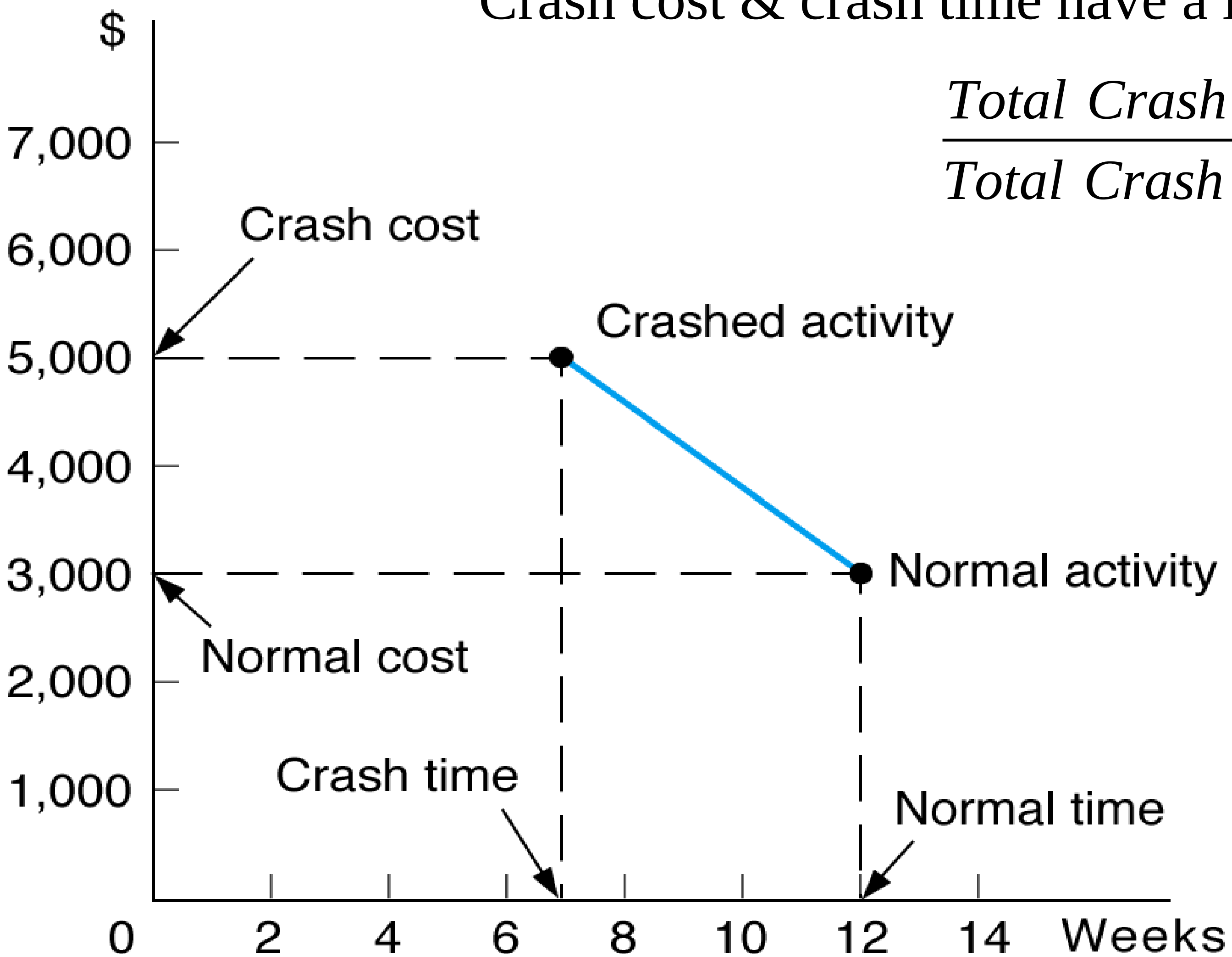
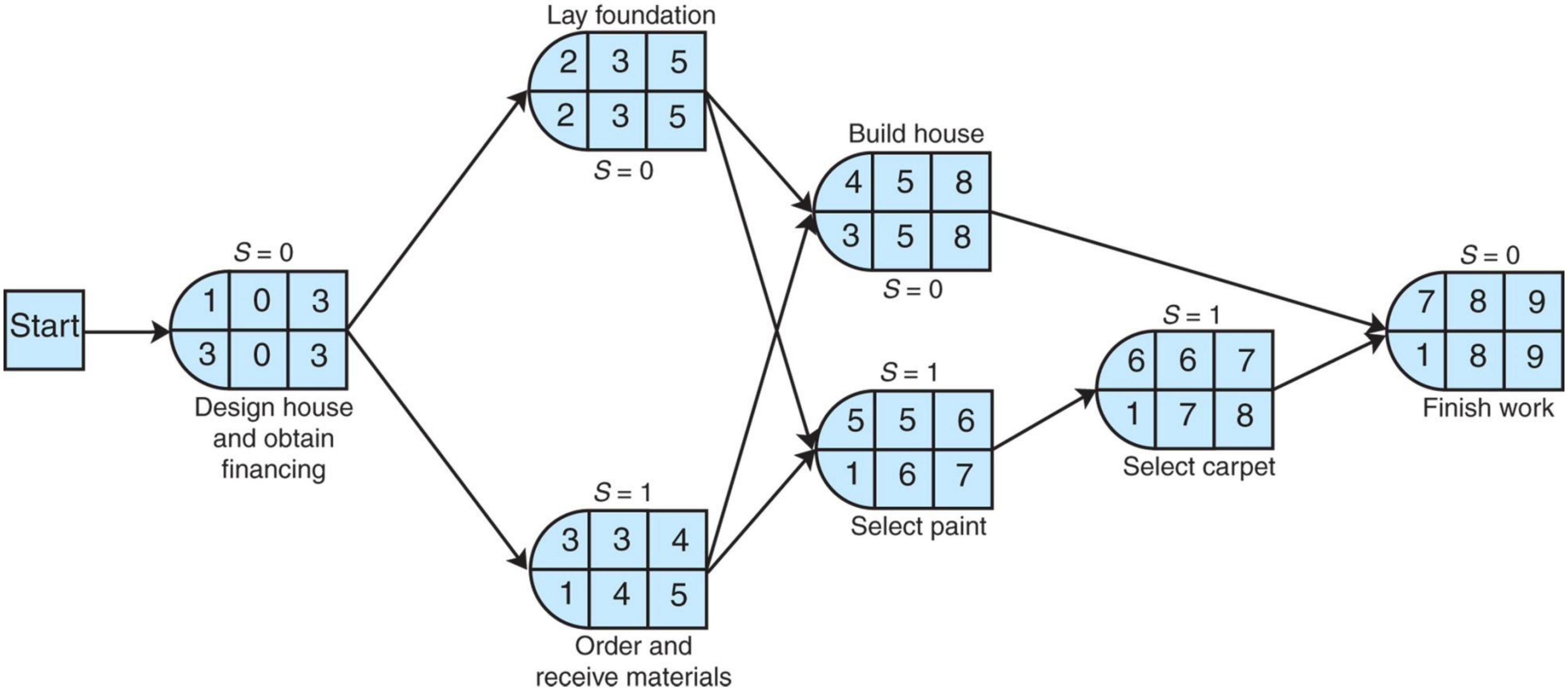


Figure 3.6

Project Crashing

Prepare data



☞ Critical path: **1 – 2 – 4 - 7**

Project Crashing

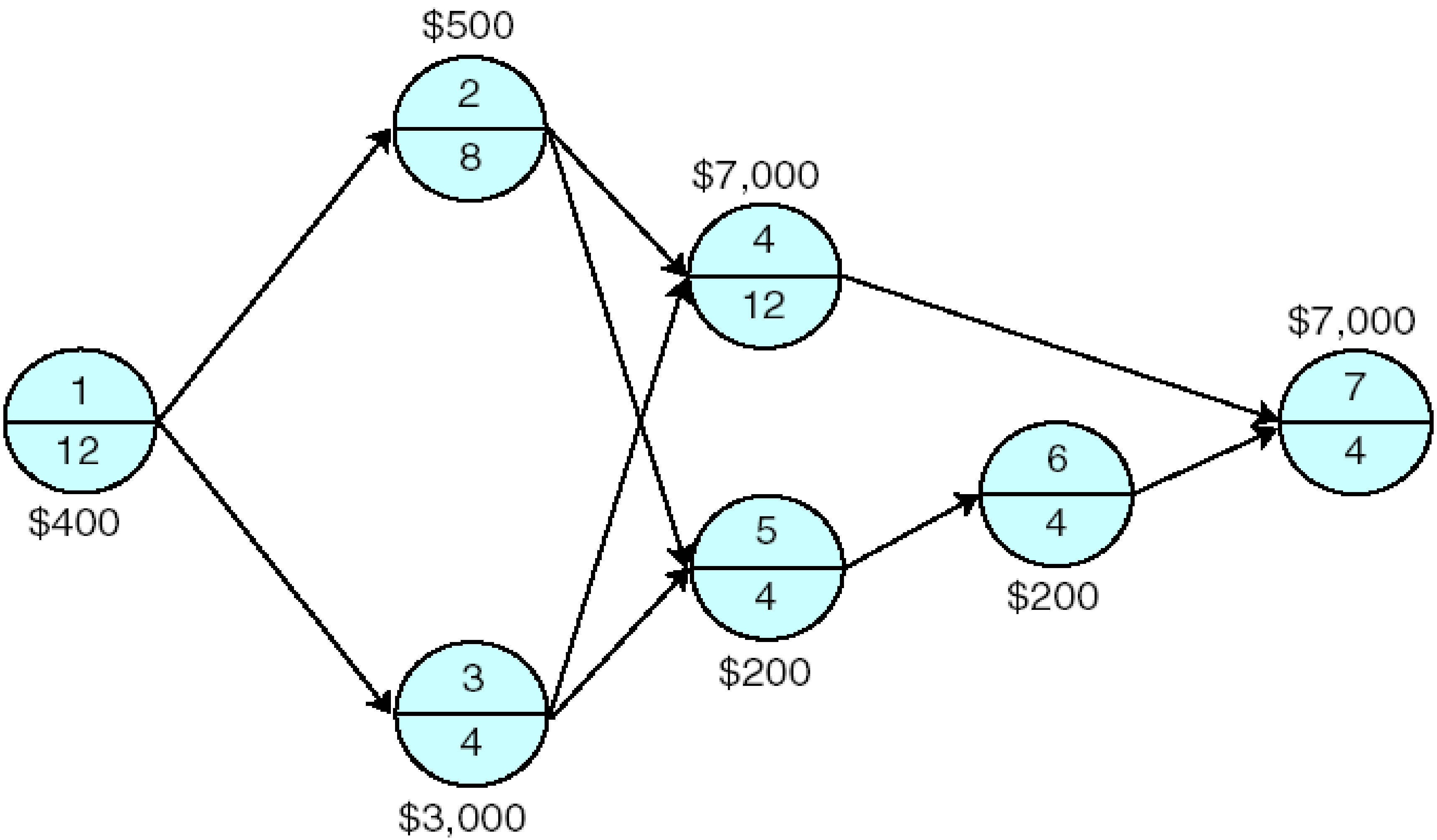


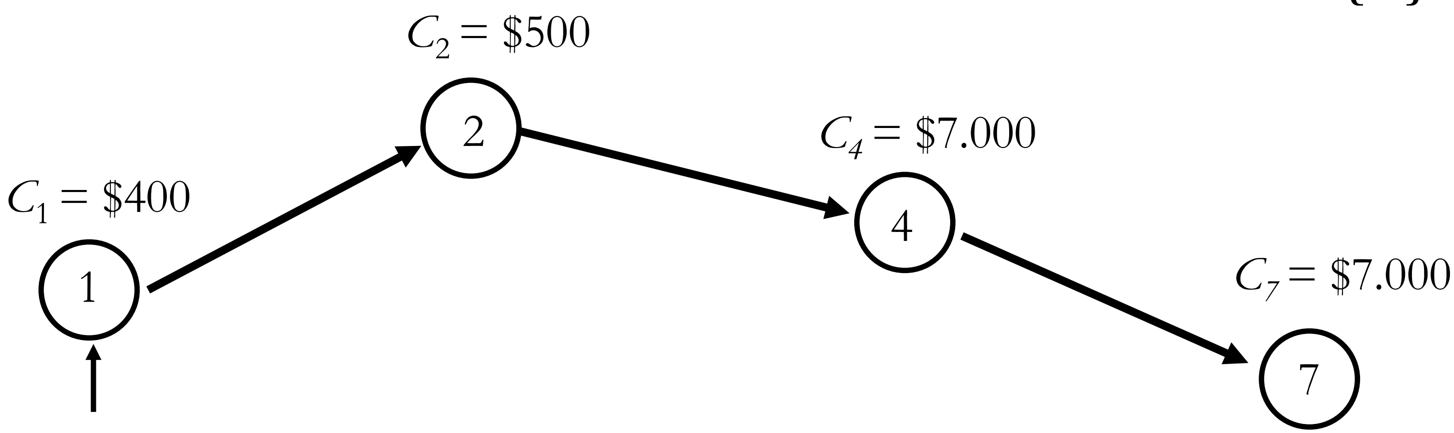
Figure 3.7. *Network with weekly crashing costs*

Project Crashing

Critical Path Subgraph (G_{cp})

Critical Path: {1},{2},{4},{7}

Minimum cost activiti: {1}



Cashing required

1. Cashing in Critical path activities
2. In Alowable cash time
3. Minimum cost activity

Project Crashing

Implement Cashing

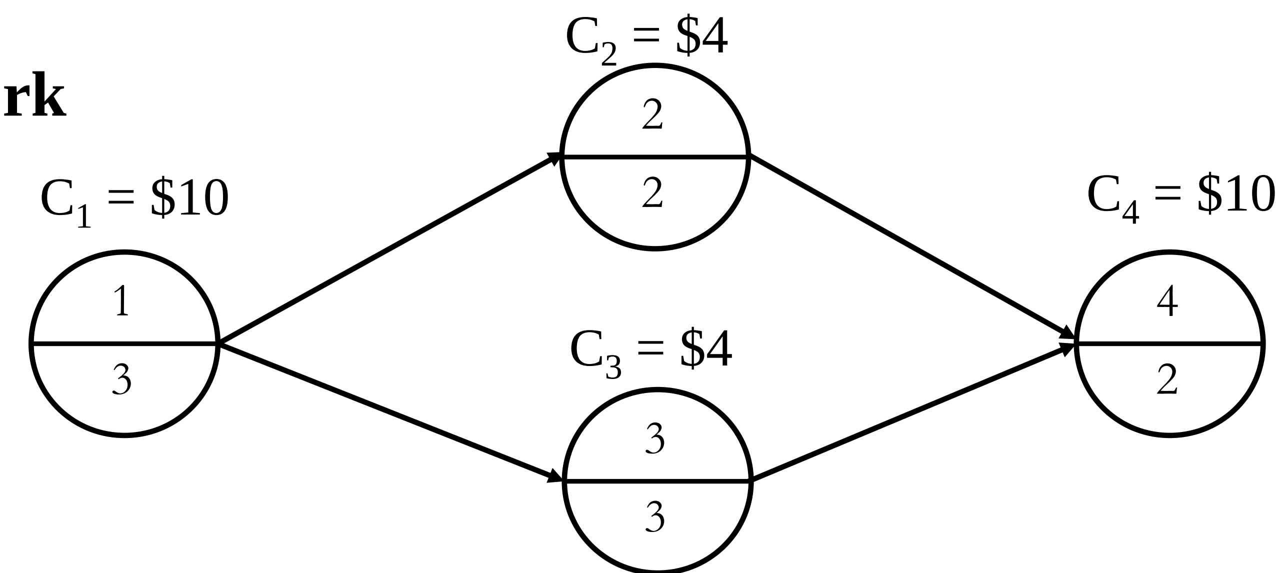
Project time	Period cost	Cum. cost	Act. 1	2	3	4	5	6	7
36	0	0							
35	400	400	1						
34	400	800	2						
33	400	1200	3						
32	400	1600	4						
31	400	2000	5						
30	500	2500	5	1					
29	500	3000	5	2					
28	500	3500	5	3					
27	7000	10500	5	3		1			
26	7000	17500	5	3		2			
25	7000	24500	5	3		3			
24	7000	31500	5	3		3			1

Project Crashing

Project data

Activity	Priorities	Normal time (weeks)	Cach time (weeks)	Normal Cost	Cash Cost	Allowable Cash time	Cask Cost per Week
1	-	3	1	120	100	2	$10
2	1	2	1	60	56	1	$4
3	1	3	1	80	72	2	$4
4	2, 3	2	1	120	110	1	$10

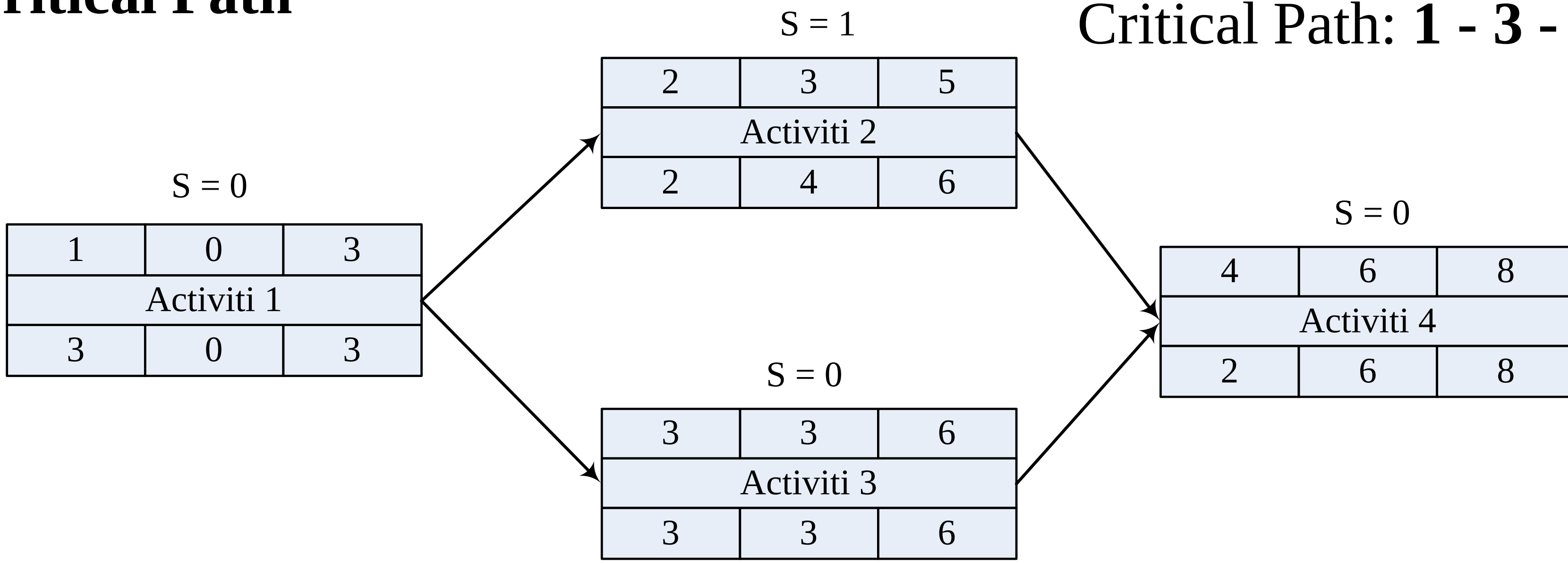
Net work



Project Crashing

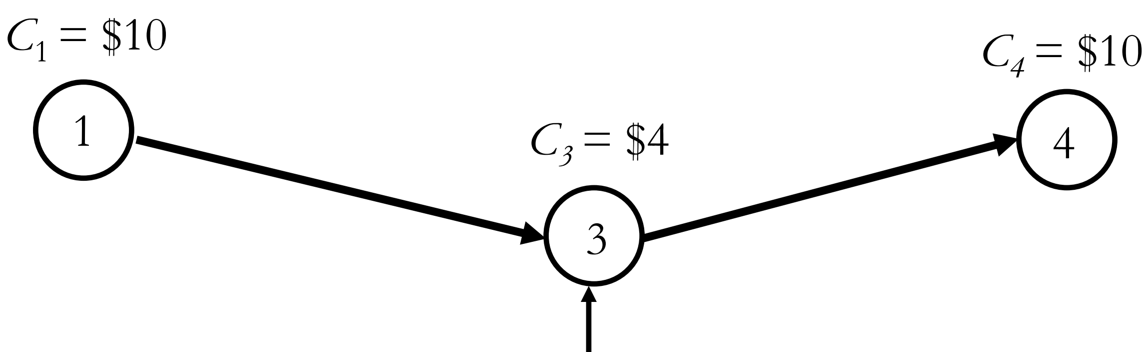
Critical Path

Critical Path: **1 - 3 - 4**



Critical Path Subgraph (G_{cp})

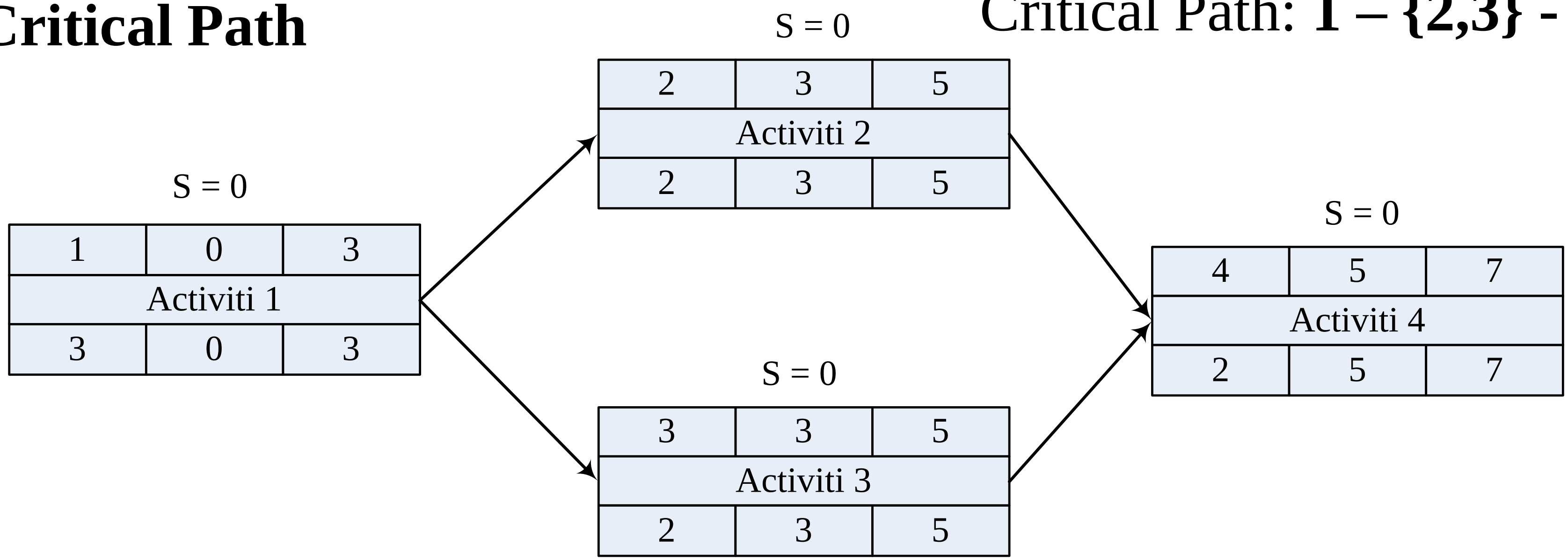
Minimum cost activiti: {3}



Project Crashing

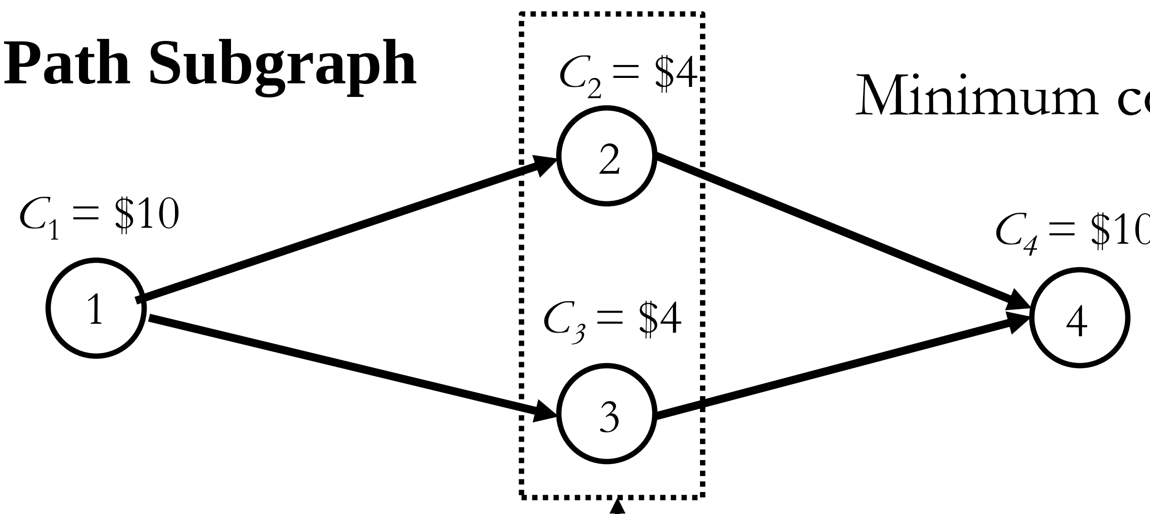
Critical Path

Critical Path: **1 – {2,3} - 4**



Critical Path Subgraph

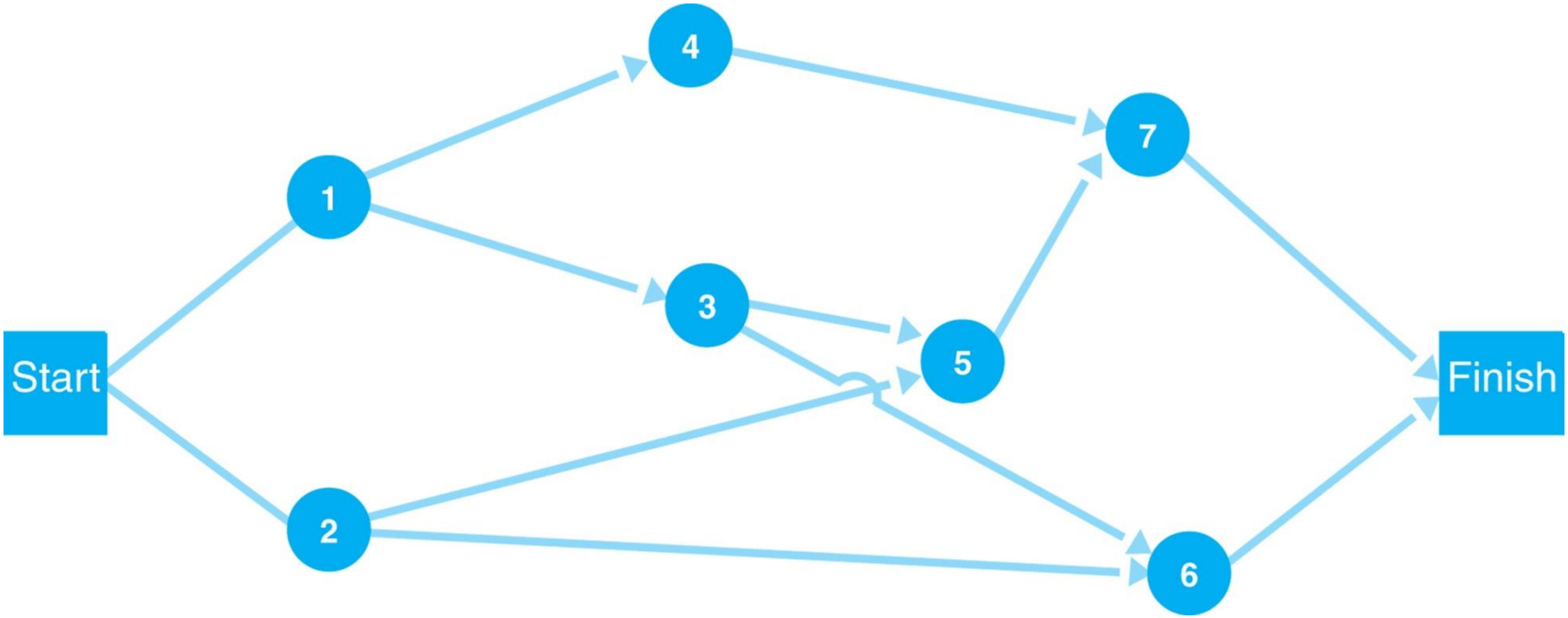
Minimum cost activiti: {2,3}



Example Problem

Problem Statement and Data

Given this network and the data on the following slide, determine the expected project completion time and variance, and the probability that the project will be completed in 26 days or less.



Example Problem

Problem Statement and Data

	Time Estimates (weeks)		
Activity	a	m	b
1	5	8	17
2	7	10	13
3	3	5	7
4	1	3	5
5	4	6	8
6	3	3	3
7	3	4	5

Example Problem

Solution

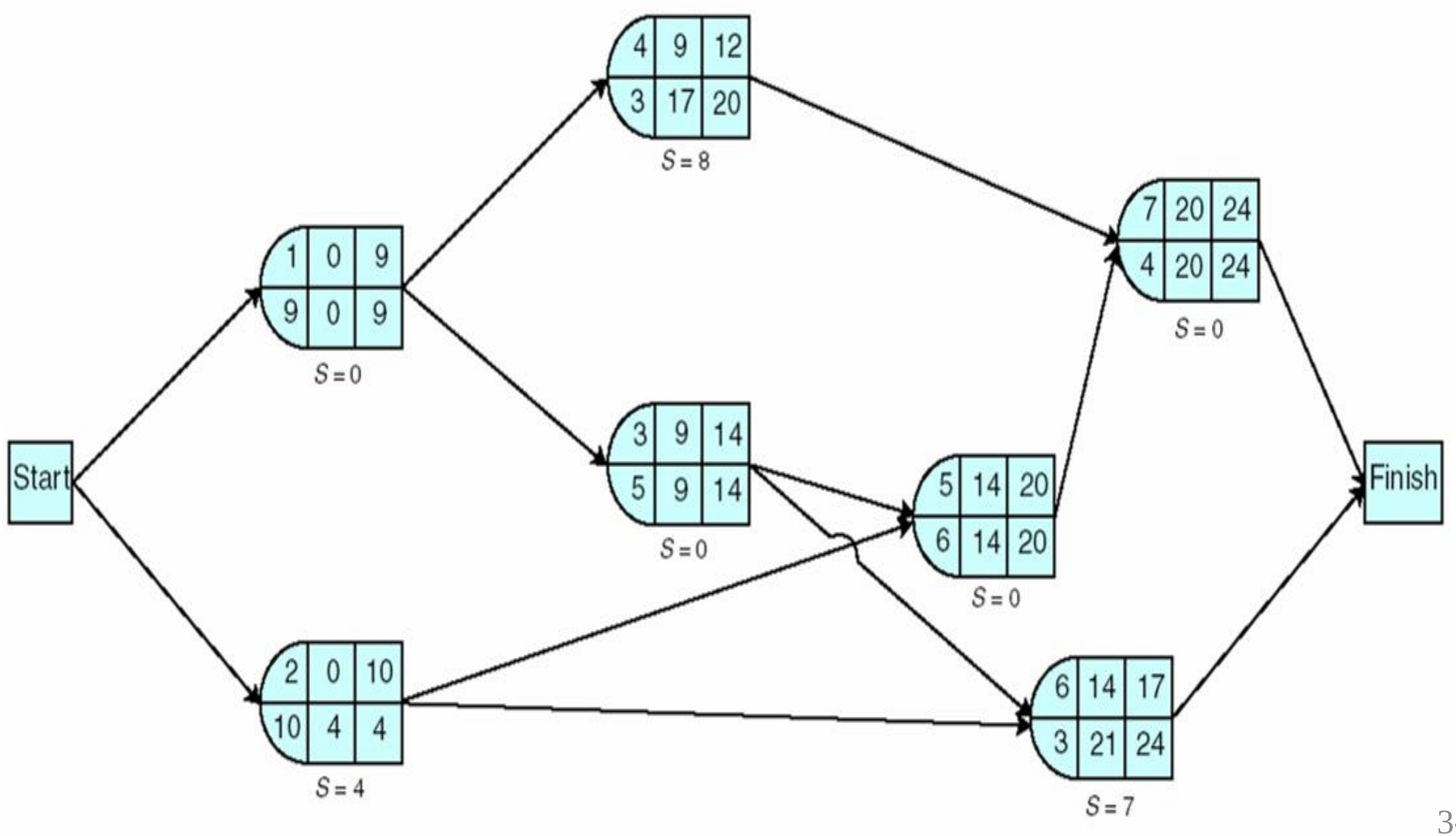
Step 1: Compute the expected activity times and variances.

$$t = \frac{a + 4m + b}{6} \qquad v = \left(\frac{b - a}{6}\right)^2$$

Activity	t	v
1	9	4
2	10	1
3	5	4/9
4	3	4/9
5	6	4/9
6	3	0
7	4	1/9

Example Problem

Step 2: Determine the earliest and latest activity times & slacks



Example Problem

Step 3: Identify the critical path and compute expected completion time and variance.

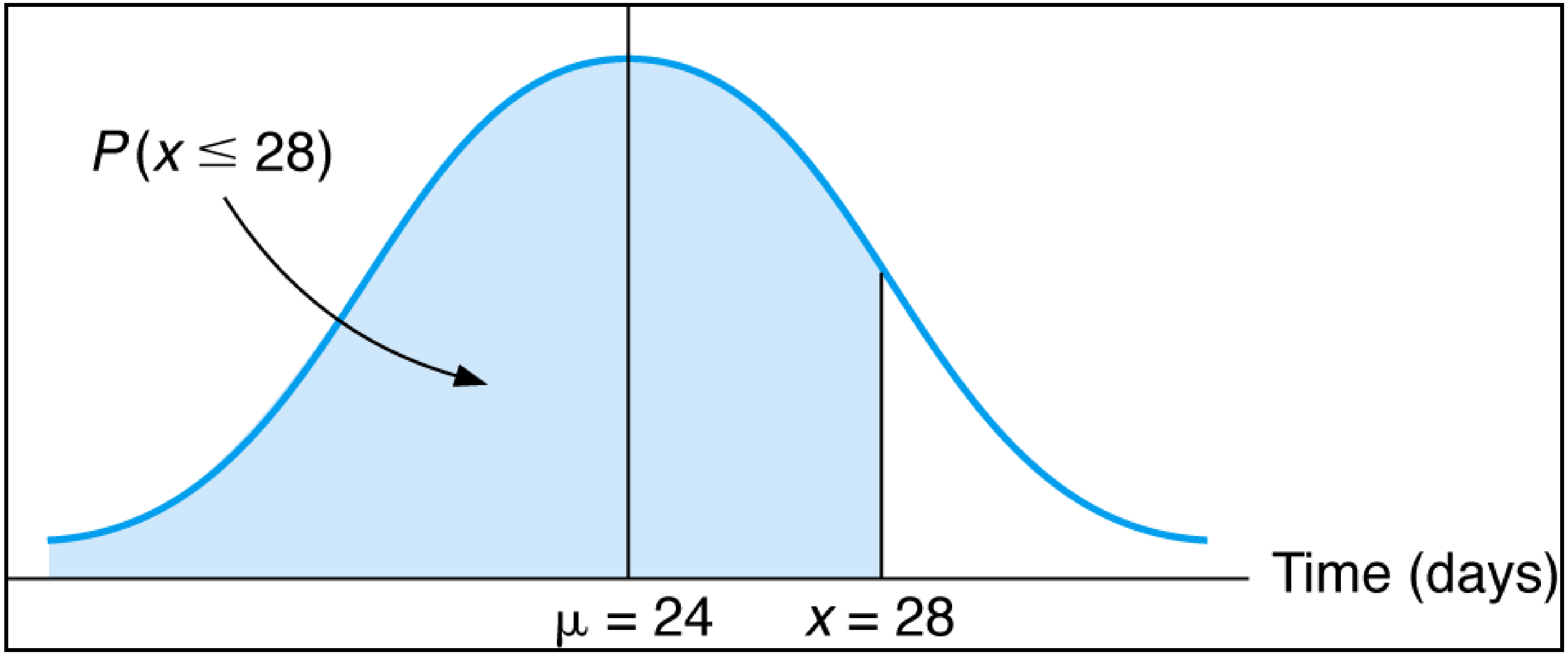
- Critical path (activities with no slack): $1 \rightarrow 3 \rightarrow 5 \rightarrow 7$
- Expected project completion time: $t_p = 9+5+6+4 = 24$ days
- Variance: $v_p = 4 + 4/9 + 4/9 + 1/9 = 5$ (days)^2

Example Problem

Step 4: Determine the Probability that the Project Will be Completed in 28 days or less ($\mu = 24$, $\sigma = \sqrt{5}$)

$$Z = (x - \mu)/\sigma = (28 - 24)/\sqrt{5} = 1.79$$

Corresponding probability from Table 3.2, is .9633 and $P(x \leq 28) = 0.9633$.



Home work

Precedence Relationships Chart

Activity	Immediate predecessor	Optimistic	Most Likely	Pessimistic
A	-	76	86	120
B	A	12	15	18
C	B	4	5	6
D	G	15	18	33
E	D	18	21	24
F	A	16	26	30
G	C, F	10	13	22
H	D	24	28	32
I	A	22	27	50
J	D, I	38	43	60

a. Find Critical Path?

b. Percentage of that we finish the project longer than mean-time 2 units?

Home work

Information of a project as follows

Activity	Immediate predecessor	Time (week)		Cost (mil.$)	
		Normal time	Crash time	Normal cost	Crash cost
A	-	6	3	100	200
B	A	7	4	50	80
C	A	2	1	150	180
D	A	5	3	200	250
E	B,C	7	3	20	40
F	B,C,D	5	4	20	40
G	C,D	5	4	60	80
H	E,F,G	3	2	30	60

a. Drawing AON project network, find critical path and time to complete the project?

b. Calculate the cost for project implementation in normal time and when crashing 5 weeks?